Angle-Leg Bench & Compound Angle Designs

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Figure 1: Angle-leg Benches

Benches with angled legs provide an extra degree of stability over standard straight leg designs. This added degree of stability can be important for smaller benches used to support heavy machinery such as saws, planers, drill presses and similar power tools. Angled legs are frequently found on such items as contractor saws and metal tables for power tools.

In an effort to make tables for some of my power tools (see pictures Figure 1), I have built several angled leg tables. These efforts eventually led me to a fuller understanding of the issues with compound miters and their effect on the design of benches with angled legs as well as other designs employing compound angles.

1 Table and Bench Stability

As somewhat of a side note, I decided to investigate how leg angle affects bench stability. The three curves shown in Figure 2 illustrate the relative effect between straight legs (0 degree), legs angled at 6 degrees and legs angled at 8 degrees for the planer bench shown in the right picture of Figure 1.

These curves represent the energy needed to displace the top of the table a given amount. The steeper and higher the curve the greater the stability. Based on this analysis, the 6^o angled leg provides a significant stability improvement over straight legs for the planer table. The trade-off is that the greater the leg angle the more the leg extends beyond the table top and the greater the chance your feet get caught in the table legs. The choice of 6^o legs for a 35 inch high bench appears to be a reasonable compromise between stability and foot room with the legs extending a little over 3.5 inches beyond the top at floor-level.

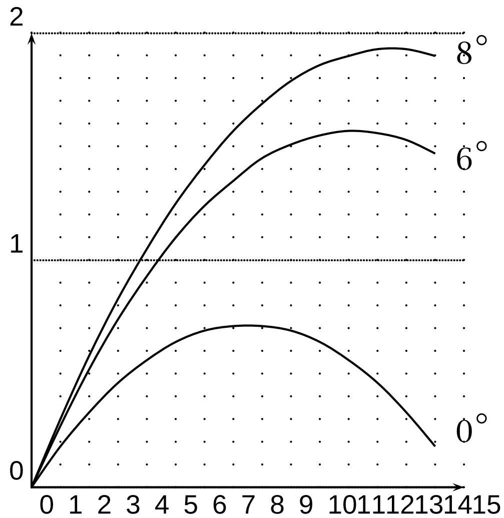
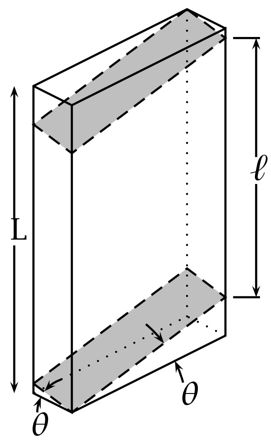


Figure 2: Stability curves for bench 35-inches high and 15-inches wide with legs angled at 0^o, 6^o and 8^o respectively.

2 Angled Leg Design Considerations

For this design the leg is angled out from the front and side rails equally by an angle θ. For purposes of cutting the ends of the legs the saw is set to cut a *compound miter* i.e. the miter gage angle set to θ and the bevel (blade angle) set to θ. The **design goal** is to cut a leg from a rectangular blank of length L such that it will extend at an angle θ from a rectangular top set at a height h from the floor. The leg blank is illustrated in Figure 3.

Table 1 gives the *leg edge length* ℓ for *bench heights* between 16 and 36 inches at leg angles of 6°, 8°, and 10°.



h	*bench top height*	
θ	*leg angle*	
ℓ	*leg edge length*	$\ell = h\sqrt{1 + 2\tan^2\theta}$
Δ	*End-waste*	$\Delta = (w + t)\tan\theta$
L	*min. stock length*	$L = \Delta + \ell$

Figure 3: Rectangular leg stock (L x w x t) with compound miter cuts illustrated in gray.

h	$\ell(6^o)$	$\ell(8^o)$	$\ell(10^o)$	h	$\ell(6^o)$	$\ell(8^o)$	$\ell(10^o)$
16	16.18	16.31	16.49	28	28.31	28.55	28.86
18	18.20	18.35	18.55	30	30.33	30.59	30.92
20	20.22	20.39	20.61	32	32.35	32.63	32.98
22	22.24	22.43	22.67	34	34.37	34.67	35.04
24	24.26	24.47	24.74	36	36.40	36.70	37.10
26	26.29	26.51	26.80				

Table 1: Leg edge length ℓ for various table heights h and leg angles.

θ	a	$tan\theta$	θ	a	$tan\theta$
4	1.0049	.0699	8	1.0196	.1405
5	1.0076	.0875	9	1.0248	.1584
6	1.0110	.1051	10	1.0306	.1763
7	1.0150	.1228	12	1.0442	.2126

Table 2: $a = \sqrt{1 + 2tan^2\theta}$ and $tan\theta$ at 1^o intervals

Table 2 lists values for $a = \sqrt{1 + 2tan^2\theta}$ and $tan\theta$ at 1° intervals between 4° and 12°. Compute the overall *minimum leg blank length* as follows:

$$L = \ell + (w + t)tan\theta$$

where ℓ is from the Table 1 or computed using the a from Table 2 as follows:

$$\ell = a \times h$$

2.1 Fitting the leg to a rectangular top

The legs of a bench require a solid support structure to hold the legs firmly in place. Typically this is a rectangular frame around the top and a cross-brace or shelf midway to the floor.

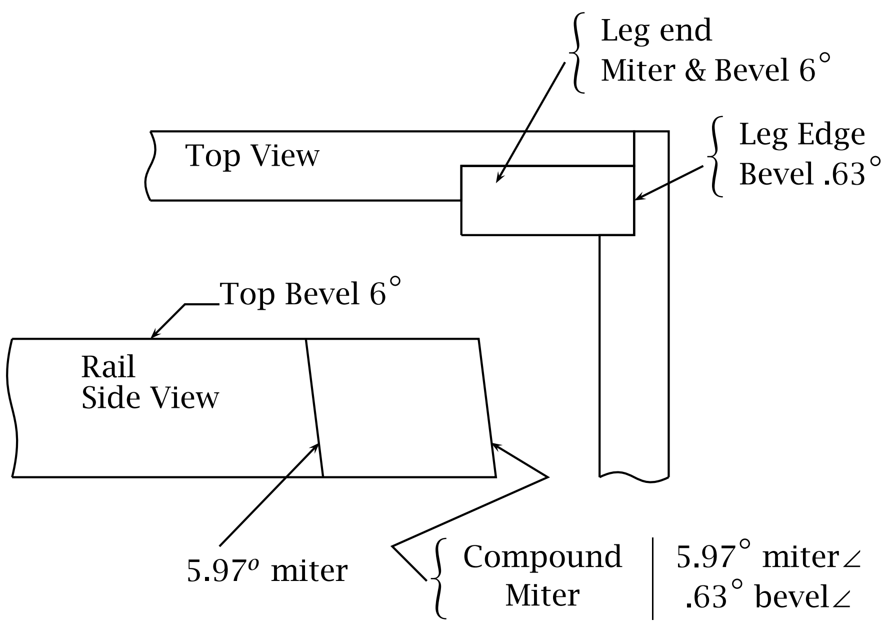


Figure 4: Top frame showing miter and bevel cuts for bench with legs angled at 6^o.

In my design I chose to use lap joints at the top that overlap the leg around the outside (refer to Figure 4). This frame defines the angle of the legs and provides alignment during assembly. Since the legs are angled, the frame must conform to the leg angles and thus compound miter joints and issues similar to those encountered with crown molding corners.

> *Note that there is a* compound miter related issue *with the legs in that they must fit in the corners formed by the top frame.*

The leg stock is typically rectangular. However the corners formed by the frame, while rectangular in the plane parallel to the floor, are somewhat greater than 90° at the angle the leg meets the frame.

To get a tight joint between the leg and the frame one must bevel the leg along the edge(s) facing out. This bevel can be just the *but-bevel* from the compound miter table in the next section …or half the but-bevel on each of the two outside edges of the leg.

3 Compound Miters

The following two formulas define respectively the *miter* and *bevel* angle cuts needed to join two boards with a compound miter joint:

$$\text{Miter-Angle} = \tan^{-1}(\sin(\theta)\tan(\alpha/2))$$

$$\text{Bevel-Angle} = \sin^{-1}(\cos(\theta)\sin(\alpha/2))$$

where:

- θ is the *slope angle* …the angle between the board face and the normal to the plane defined by the axis of the two boards.
- α is the *bend angle* …the angle between the axis of the two boards (see Figure 5).

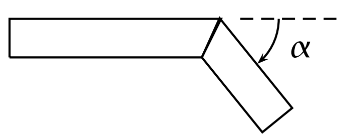


Figure 5: Compound Miter Joint illustrating "*bend angle α.*"

> **Note:** *The definition of some angles as used in this discussion including θ and the miter gage angle are different than some other sources. These differences are reflected in the equations and tables presented here.*

α for some typical geometric shapes	
rectangle (4–sides)	$\alpha = 90^\circ$
hexagon (6–sides)	$\alpha = 60^\circ$
octagon (8–sides)	$\alpha = 45^\circ$
N–sides	$\alpha = 360/N$

θ	*Slope angle* [1]
Miter	Miter gage angle (0° = standard right angle cut) [2]
Bevel	Saw blade angle *Miter-Joint* (0° = standard right angle cut)
But-bvl	Saw blade angle *But-Joint* ($\alpha = 90^\circ$)
δ	bevel-joint *compound-angle correction* $\delta = \alpha - 2(\text{bevel})$

	4-Side	$\alpha = 90.00$	
θ	Miter	Bevel	but-bvl(δ)
4	3.99	44.86	0.28
5	4.98	44.78	0.44
6	5.97	44.69	0.63
7	6.95	44.57	0.85
8	7.92	44.45	1.11
9	8.89	44.30	1.40
10	9.85	44.14	1.73
15	14.51	43.08	3.84
20	18.88	41.64	6.72
25	22.91	39.86	10.29
30	26.57	37.76	14.48
35	29.84	35.40	19.21
40	32.73	32.80	24.40
45	35.26	30.00	30.00

	6-Sides	$\alpha = 60$	
θ	Miter	Bevel	δ-bevel
4	2.31	29.92	0.16
5	2.88	29.87	0.25
6	3.45	29.82	0.36
7	4.02	29.75	0.49
8	4.59	29.68	0.64
9	5.16	29.59	0.81
10	5.73	29.50	1.00
15	8.50	28.88	2.24
20	11.17	28.02	3.95
25	13.71	26.95	6.11
30	16.10	25.66	8.68
35	18.32	24.18	11.64
40	20.36	22.52	14.96
45	22.21	20.70	18.59

	8-Sides	$\alpha = 45$	
θ	Miter	Bevel	δ-bevel
4	1.66	22.44	0.12
5	2.07	22.41	0.18
6	2.48	22.37	0.26
7	2.89	22.32	0.35
8	3.30	22.27	0.46
9	3.71	22.21	0.58
10	4.11	22.14	0.72
15	6.12	21.69	1.61
20	8.06	21.08	2.85
25	9.93	20.29	4.41
30	11.70	19.35	6.29
35	13.36	18.27	8.46
40	14.91	17.05	10.91
45	16.32	15.70	13.60

	12-Sides	$\alpha = 30$	
θ	Miter	Bevel	δ-bevel
4	1.07	14.96	0.07
5	1.34	14.94	0.12
6	1.60	14.92	0.17
7	1.87	14.89	0.23
8	2.14	14.85	0.30
9	2.40	14.81	0.38
10	2.66	14.77	0.47
15	3.97	14.48	1.04
20	5.24	14.08	1.85
25	6.46	13.57	2.87
30	7.63	12.95	4.09
35	8.74	12.24	5.52
40	9.77	11.44	7.13
45	10.73	10.55	8.91

[1] Note: The definition of θ or Slope angle varies from conventions used by some other compound miter discussions.

[2] Note: This definition of miter angle varies from some conventional usage.

4 Eight-Sided Basket with Sloping Sides

As a further example of the effects of compound angle joints consider the design of an eight-sided basket with 5^o sloping sides as pictured in Figure 6.

Figure 6: Eight-Sided Basket with 5^o Sloping Sides

One approach is to use a compound miter joint with angles from the *compound miter tables.* Glue-up with be complicated by the fact that there is nothing to hold the registration of the eight slats. An 8-sided glue-joint bit (e.g. MLCS #7839 and others) provides an edge that helps register the slats during glue-up. This bit is however ground for 8-sided designs with 0^o slope sides. The compound-angle correction δ for 5^o slope sides is relatively small ($.18^o$) and could be ignored provided the error is not allowed to accumulate i.e. evenly distributed at each joint. Note that the accumulated error is $8x.18 = 1.44^o$. Larger slopes could however present problems.

The alternative is to route the slats with the slat angled up $.18^o$ …*provided the 8-sided glue-joint bit is ground so that the slats can be routed with the outside face up since the object is to increase the angle between slats by* $.18^o$*.*

My solution to angling the slat $.18^o$ is to place enough layers of blue painters tape near the back-edge of the slat to achieve the desired angle …in my case with the tape 3 inches from the edge to be routed. For $.18^o$ at 3-inches this requires a tape thickness $3sin(\delta) = 3sin(.18) = .0094$ or nearly 1/100th inch. Blue tape measures about .0033 so it takes about 3-layers at 3-inches to get $.18^o$.

4.1 Multi-Slat Glue-up

Even with the glue-joint bit, glue-up is no picnic. Juggling eight slats with glue doesn't work too well. My approach was to divide and conquer by gluing pairs of slats together first and then the four glued segment-pairs. This however requires that the pairs be glued at the precise angle required by the compound joint angle $(180 - 2 \times \text{bevel})$.

To hold and clamp pairs of slats I designed an adjustable cradle to hold and clamp slat-pairs for glue-up. A picture of the cradle is shown in Figure 7. Pairs of 5.5 inch-wide support blocks are cut at an the angles of 22.5^o and

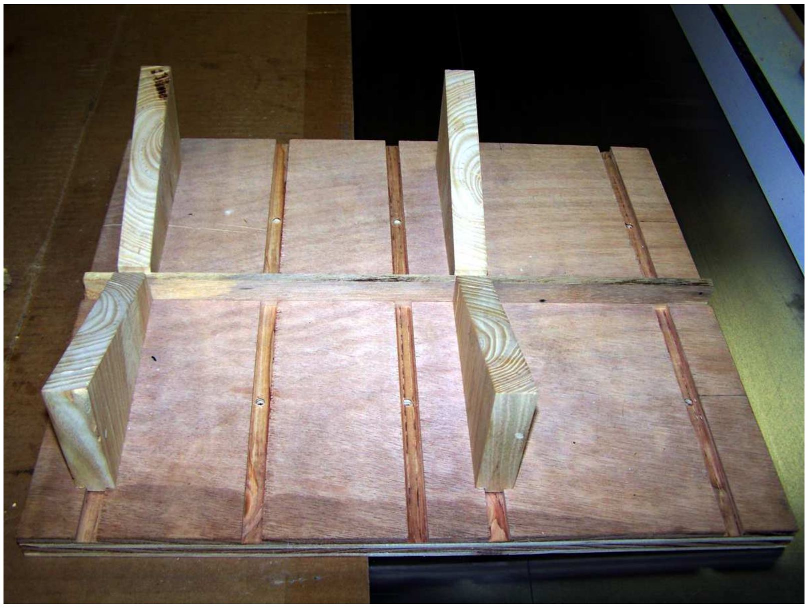


Figure 7: Glue-up Cradle.

$22.5 - \delta = 22.32^o$ respectively and mounted vertically in the cradle base. The support blocks are positioned by a 3/8 wide dados in the base and a 1/4-inch spacer running across the base center. Each block is held in place by a single pan-head screw recessed in the bottom of the cradle base. Spacing of the support blocks can be changed by placing them in different dado slots. Different angles are achieved by machining new support blocks. Clamping the glue-up is done with one or more straps around the cradle. When tightened the strap forces the slats down and in toward the center of the jig.

The purpose of the dados and center spacer is to provide accurate registration of the support blocks. The screw hole in the base is slightly over-sized so the support blocks can be positioned against the center spacer before tightening the support anchor screws. The center spacer is also 1/4 inch shorter than the edge of the block to accommodate glue squeeze-out.